Statistics

Lecture 21

Feb 19-8:47 AM

SG 22 & SG 23

Estimating Parameters

Samples ⟷ Statistic

Populations ⟷ Parameters

To estimate Parameters, we use similar statistic.

To estimate | We use

$\underbrace{\text{Population Proportion}}_{p}$ | $\underbrace{\text{Sample Proportion}}_{\hat{p}}$ p_hat

$\underbrace{\text{Population Mean}}_{\mu}$ | $\underbrace{\text{Sample Mean}}_{\bar{x}}$

Sample Prop

Point-estimate

Point estimate for

Pop. Prop. p is $\hat{p}$

Pop. Mean μ is $\bar{x}$ ← Sample Mean

Nov 13-12:16 PM

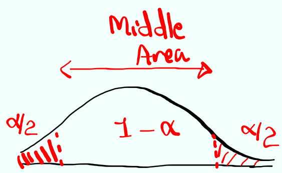
When estimating a parameter, the answer is a $\boxed{\text{range of Values.}}$

Confidence Interval

Every Conf. interval Comes with $\underbrace{\text{Confidence level.}}_{\text{Middle area}}$

$(1-\alpha)\cdot 100\%$

$0<\alpha<1$



$\frac{\alpha}{2}$ is the area of each tail.

When C-level not given ⟹ Use 95% C-level.

When α Significance level not given ⟹ Use .05

Nov 13-12:23 PM

Confidence Interval for $\underbrace{\text{Population Proportion}}_{P}$

Point-estimate

$$\hat{P} - E < P < \hat{P} + E$$

Sample Proportion

Margin of error

$$\hat{P} = \frac{x}{n}$$

x ← # of favorable responses

n ← Sample Size

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{P}\,\hat{q}}{n}}$$

$$\hat{q} = 1 - \hat{P}$$

Critical Value for $(1-\alpha)\cdot 100\%$ C-level.

Nov 13-12:29 PM

I randomly selected 100 college students and 80 of them voted in presidential election.

$n = 100 \quad x = 80$

$\hat{p} = \frac{x}{n} = \frac{80}{100} = \boxed{.8}$

$\hat{q} = 1 - \hat{p} = 1 - .8 = \boxed{.2}$

C-level: .98

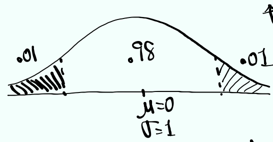
$\hat{p} - E < p < \hat{p} + E$

Let's construct $\boxed{98\%}$ $\boxed{\text{Conf. level}}$ for prop. of all students that voted.

$.8 - .09 < p < .8 + .09$

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.326 \cdot \sqrt{\frac{(.8)(.2)}{100}} \approx \boxed{.09}$

$\boxed{.71 < p < .89}$ ✓



$Z_{\alpha/2} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

We are 98% confident that between 71% & 89% of all college students voted in presidential election.

STAT | TESTS | 1-PropZInt

$x = 80$
$n = 100$
C-level: .98
Calculate

$.706 < p < .893$

$\boxed{.71 < p < .89}$

Nov 13-12:33 PM

I surveyed 250 college students, 40 were smokers.

$x = 40$
$n = 250$

$\hat{p} = \frac{x}{n} = \frac{40}{250} = .16$ Point-estimate

$\hat{q} = 1 - \hat{p} = .84$

Conf. interval for prop. of all students that smoke using 90% C-level.

$\hat{p} - E < p < \hat{p} + E$

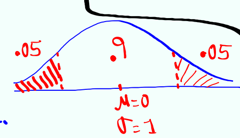
$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$.16 - .04 < p < .16 + .04$

$= 1.645 \cdot \sqrt{\frac{(.16)(.84)}{250}} \approx \boxed{.04}$

$\boxed{.12 < p < .20}$

We are 90% confident that between 12% & 20% of all students smoke.



STAT | TESTS

$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

1-PropZInt

$x = 40$
$n = 250$
C-level: .9
Calculate

$.12 < p < .20$

$E = \frac{.20 - .12}{2} = \frac{.08}{2} = \boxed{.04}$ ✓

$\hat{p} = \frac{.20 + .12}{2} = \frac{.32}{2} = \boxed{.16}$

Nov 13-12:48 PM

I surveyed 125 college students and 8% of them were on diet to lose weight.

$n = 125$ $\quad \hat{p} = \frac{x}{n}$

$\hat{p} = .08$ $\quad x = n\hat{p} = 125(.08) = 10$

if decimal, Round-up

Find Conf. interval for prop. of all students that have been on diet to lose weight using 99% C-level.

1 - Prop Z Int
$x = 10$
$n = 125$
C-level: .99
Calculate

$.02 < p < .14$

we are 99% confident that between 2% & 14% of all students have been on diet to lose weight.

$$E = \frac{.14 - .02}{2} = .06$$

$$\hat{p} = \frac{.14 + .02}{2} = .08$$

Nov 13-12:59 PM

I surveyed 240 voters, and 72% of them trusted the result of last presidential election.

$n = 240$ $\quad \Rightarrow x = n\hat{p} = 240(.72) = 172.8$ $\quad \boxed{x = 173}$

$\hat{p} = .72$ $\quad$ if decimal $\rightarrow$ Round-up

No C-level

Find $\boxed{\text{Conf. interval}}$ for the prop. of all voters that trusted the outcome of the election.

$\rightarrow$ use .95

1 - Prop Z Int
$x = 173$
$n = 240$
C-level: .95
Calculate

$\boxed{.66 < p < .78}$

$$E = \frac{.78 - .66}{2} = \boxed{.06}$$

$$\hat{p} = \frac{.78 + .66}{2} = \boxed{.72}$$

Nov 13-1:07 PM

Given $.125 < p < .275$

1) $E = \frac{.275 - .125}{2} = \boxed{.075}$

2) $\hat{p} = \frac{.275 + .125}{2} = \boxed{.2}$

Given $n = 184$ $\hat{p} = .35$

1) $\hat{q} = 1 - \hat{p} = \boxed{.65}$

2) $x = n\hat{p} = 184(.35) = 64.4 \rightarrow \boxed{x = 65}$

if decimal → Round-up

Nov 13-1:14 PM

Estimating Population Mean μ

$$\bar{x} - E < \mu < \bar{x} + E$$

Point-estimate

Margin of error

Case I: σ Known

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

TI:

STAT **TESTS**

Z Interval

inpt: Stats

Nov 13-1:37 PM

Given: $n = 25$, $\bar{x} = 88$, $\boxed{\sigma = 10}$

99% C-level

Find Conf. interval for population mean.

σ Known

$\bar{x} - E < \mu < \bar{x} + E$

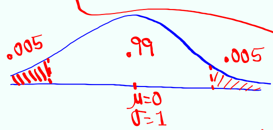
$88 - 5 < \mu < 88 + 5$

$E = Z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$

$= 2.576 \cdot \dfrac{10}{\sqrt{25}} \approx \boxed{5}$

$\boxed{83 < \mu < 93}$ ✓

We are 99% Confident that Pop. mean is between 83 & 93.



$Z_{\alpha/2} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

σ Known → Z Interval

inpt: Stats

$\sigma = 10$

$\bar{x} = 88$

$n = 25$

C-level: .99

Calculate

$\boxed{83 < \mu < 93}$

$E = \dfrac{93 - 83}{2} = 5$

$\bar{x} = \dfrac{93 + 83}{2} = 88$

Nov 13-1:41 PM

A sample of 32 College Students had a mean age of 30 Yrs old.

$n = 32$

$\bar{x} = 30$

It is Known that Standard deviation of ages of all Students is 10 Yrs. $\sigma = 10$

Find Conf. interval for the mean age of all Students.

No C-level

Use .95

$\bar{x} - E < \mu < \bar{x} + E$

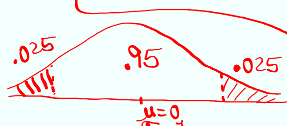
$30 - 3 < \mu < 30 + 3$

$\boxed{27 < \mu < 33}$

$E = Z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$

σ Known

$E = 1.960 \cdot \dfrac{10}{\sqrt{32}} \approx 3$



$Z_{\alpha/2} = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

Z Interval

inpt: Stats

$\sigma = 10$

$\bar{x} = 30$

$n = 32$

C-level: .95

Calculate

Nov 13-1:52 PM

Estimating Population Mean μ

$$\bar{x} - E < \mu < \bar{x} + E$$

Point-estimate ($\bar{x}$) | Margin of error (E)

Case I: σ Known

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

TI:

$\boxed{\text{STAT}}$ $\boxed{\text{TESTS}}$

$\boxed{\text{Z Interval}}$

inpt: $\boxed{\text{Stats}}$

Case II: σ Unknown

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \qquad df = n-1$$

TI

$\boxed{\text{STAT}}$ $\boxed{\text{TESTS}}$

$\boxed{\text{T Interval}}$

inpt: $\boxed{\text{Stats}}$

Nov 13-1:37 PM

A Sample of 15 exams had a mean of 86 and Standard dev. of 12.

$n = 15$
$\bar{x} = 86$
$s = 12$

C-level: .9

Find $\boxed{\text{90\% Conf. interval}}$ for the mean of all exams.

σ unknown

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$= 1.761 \cdot \frac{12}{\sqrt{15}} \approx 5$$

$\bar{x} - E < \mu < \bar{x} + E$

$86 - 5 < \mu < 86 + 5$

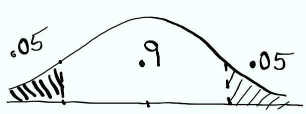
$\boxed{81 < \mu < 91}$

$\boxed{\text{T Interval}}$

inpt: $\boxed{\text{Stats}}$

$\bar{x} = 86$
$s = 12$
$n = 15$
C-level: .9

$\boxed{\text{Calculate}}$



$\mu = 0$
σ unknown
$df = n - 1 = 14$

$t_{\alpha/2} = \text{invT}(.95, 14) = 1.761$

Nov 13-2:04 PM

10 randomly selected nurses had a mean monthly salary of \$7750 with standard deviation of \$380.

df=9 → $n=10$ $\bar{x}=7750$ $S=380$

σ - unknown

Find $\boxed{\text{Conf. interval}}$ for the mean salary of all nurses. → NO C-level use .95

T Interval

inpt: $\boxed{\text{Stats}}$

$\bar{x}=7750$

$S=380$

$n=10$

C-level: .95

$\boxed{\text{Calculate}}$

$\boxed{7478 < \mu < 8022}$

$E = \frac{8022-7478}{2} = \boxed{272}$

$\bar{x} = \frac{8022+7478}{2} = \boxed{7750}$

Nov 13-2:12 PM

I randomly selected 10 gas stations. Here are prices of gasoline /ga.

4.25	4.65	3.85	4.10
4.15	4.75	4.35	3.90
	4.25	4.25	

Store in L1

use $\boxed{\text{1-Var Stats}}$

$\bar{x}=4.25$

$S=.29$

$n=10$

Find 98% Conf. interval for the mean gas price of all gas stations.

σ unknown

T Interval

inpt: $\boxed{\text{Stats}}$

$\bar{x}=4.25$

$S=.29$

$n=10$

C-level: .98

$\boxed{\text{Calculate}}$

$3.99 < \mu < 4.51$

$E = \frac{4.51-3.99}{2} = \boxed{.26}$

$\bar{x} = \frac{4.51+3.99}{2} = \boxed{4.25}$

Nov 13-2:20 PM